Module 4

- **Discrete Random Variable (DRV) ↓**
- **Bernoulli's Random Variable**

$$P(1) = P\{X = 1\} = p$$
$$P(0) = P\{X = 0\} = (1 - p)$$
where, $X = 1$ is a success
& $X = 0$ is a failure

Only for 1 outcome.

- **Binomial Random Variable**

$$P(i) = C(n, i) \,.\, p^{i} \,.\, (1 - p)^{n - i}$$

For finite outcomes.

- **Expectat^{n}**

$$E[X] = np$$

- **Variance**

$$Var(X) = (1 - p) \,.\, E[X]$$

- **Poisson's Random Variable**

$$P(i) = P\{X = i\} = \frac{e^{-\lambda}\lambda^{i}}{i!}$$

• It's a DRV with λ factor.
• For uncountable outcomes.
• X= 0, 1, 2, 3, ...

- **Expectat^{n} & Variance**

$$Var(X) = \lambda = E[X]$$

- **N.B.**

  Poisson's distribut^{n} is a limiting case of Binomial distribut^{n}.

- **Continuous Random Variable (CRV)↓**
- **Probability Density Funct^{n} (PDF) of CRV**

$$P\{X \in B\} = \int_{B} f(x)dx$$

- **Property**

$$P\{X \in (-\infty, \infty)\} = \int_{-\infty}^{\infty} f(x)dx = 1$$

- **Expected value or Mean of a CRV**

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx$$

- **Proposit^{n}**

$$E[g(x)] = \int_{-\infty}^{\infty} \{g(x)f(x)\}dx$$

- **Relat^{n} between CDF & PDF of a CRV**

$$F(a) = P\{X \leq a\} = P\{X \in (-\infty, a]\} = \int_{-\infty}^{a} f(x)dx$$

$$\frac{d}{da}F(a) = f(a)$$

Here, ***F*** is the CDF of (X) & ***f*** is the PDF of (X).

- **Uniform Random Variable (URV)**

- PDF of URV

$$f(x) = \begin{cases} \frac{1}{\beta - \alpha} & ,\alpha < x < \beta \\ 0 & ,otherwise \end{cases}$$

- CDF of URV

$$F(a) = \begin{cases} 0 & ,a < \alpha \\ \frac{a - \alpha}{\beta - \alpha} & ,\alpha < a < \beta \\ 1 & ,a > \beta \end{cases}$$

- N.B.

$$P\{c \le X \le d\} = \int_c^d \frac{1}{b - a} dx = \frac{d - c}{b - a}$$

where, (α,β)=(a,b)

- Expectat^n

$$E[X] = \frac{\beta + \alpha}{2}$$

- Variance

$$Var(X) = \frac{(\beta - \alpha)^2}{12}$$

- Normal or Gaussian or Laplace Random Variable (NRV)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \; x \in (-\infty, \infty)$$

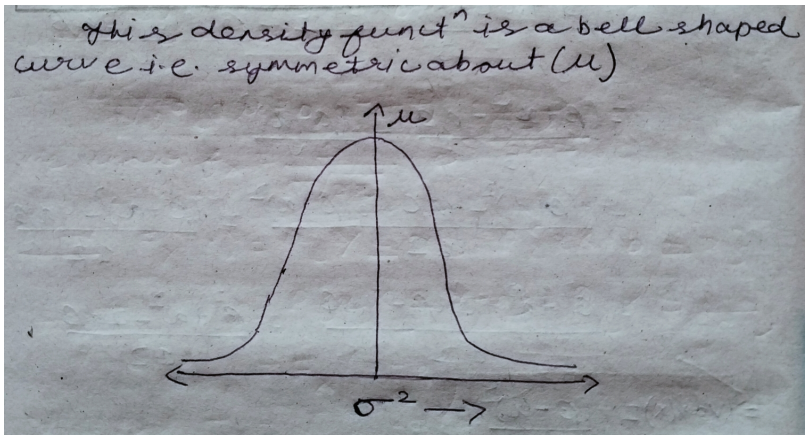
- Expectat^n

$$E[X] = \mu$$

- Variance

$$Var(X) = \sigma^2$$

- N.B.



- Std. Normal Random Variable

$$Z = \frac{X - \mu}{\sigma}$$

- Expectat^n

$$E[Z] = \mu = 0$$

- Variance

$$Var(Z) = \sigma^2 = 1$$

- CDF of Std. NRV

$$\varphi(x) = P\{Z \leq x\} = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz$$

- **N.B.**

$$\varphi(-x) = 1 - \varphi(x)$$

- **Exponential Random Variable (ERV)**

  It's a CRV with λ factor.

  - **PDF of ERV**

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ,x \geq 0 \\ 0 & ,x < 0 \end{cases}$$

  - **CDF of ERV**

$$F(a) = P\{X \leq a\} = 1 - e^{-\lambda a}$$

  - **Expectat^n**

$$E[X] = \frac{1}{\lambda}$$

  - **Variance**

$$Var(X) = \frac{1}{\lambda^2} = (E[X])^2$$

- **Discrete Distribut^n ↓**
- **Hypergeometric Distribut^n**

$$P(k) = P\{X = k\} = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

- N= Population Size
- K= No. of objects with success feature
- n= No. of trials
- k= No. of successes

- Expectat^n

$$E[X] = \frac{nK}{N}$$

- Variance

$$Var(X) = E[X]\left(\frac{N-K}{N}\right)\left(\frac{N-n}{N-1}\right)$$

- **N.B.**

Binomial distribut^n is a limiting case of Hypergeometric distribut^n.

- **Bivariate distribut^n (Jointly distributed random variable) ↓**
- **Continuous Case**
  - **CDF**
    - **Joint CDF of (X) & (Y)**

$$F_{XY}(a, b) = P\{X \le a, Y \le b\} = \int_{-\infty}^{b}\int_{-\infty}^{a} f(x, y)dxdy$$

where, -∞ < a,b < ∞

    - **Marginal CDF of (X)**

$$F_X(a) = P\{X \le a\} = P\{X \le a, Y < \infty\} = F_{XY}(a, \infty)$$

    - **Marginal CDF of (Y)**

$$F_Y(b) = P\{Y \le b\} = P\{X < \infty, Y \le b\} = F_{XY}(\infty, b)$$

  - **PDF**

- Joint PDF of (X) & (Y)

$$P\{X \in A,\ Y \in B\} = \int_B \int_A f(x, y)dxdy$$

- Marginal PDF of (X)

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy$$

- Marginal PDF of (Y)

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx$$

- Discrete Case
  - Joint PMF of (X) & (Y)

$$P_{XY}(a, b) = P\{X = a,\ Y = b\}$$

  - N.B.

$$\sum_{(x, y) \in R_{XY}} P_{XY}(x, y) = 1$$

  - Marginal PMF of (X)

$$P_X(x) = \sum_{y_j \in R_Y} P_{XY}(x, y_j)$$

  - Marginal PMF of (Y)

$$P_Y(y) = \sum_{x_i \in R_X} P_{XY}(x_i, y)$$

- Independent Random Variables

$$P\{X \in A,\ Y \in B\} = P\{X \in A\} \,.\, P\{Y \in B\}$$

$$P\{X \le a,\ Y \le b\} = P\{X \le a\} \,.\, P\{Y \le b\}$$

- For Joint CDF (Continuous)

$$F_{XY}(x, y) = F_X(x) \,.\, F_Y(y)$$

- For Joint PDF (Continuous)

$$f_{XY}(x, y) = f_X(x) \,.\, f_Y(y)$$

- For Joint PMF (Discrete)

$$P_{XY}(x, y) = P_X(x) \,.\, P_Y(y)$$

- Expectat^n of Bivariate Distribut^n

$$E[X] = \Sigma x P_X(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$E[Y] = \Sigma y P_Y(y) = \int_{-\infty}^{\infty} y f(y) dy$$

- Variance of Bivariate Distribut^n

$$Var(X) = E[X^2] - (E[X])^2$$

$$Var(Y) = E[Y^2] - (E[Y])^2$$